

با یاد او

سری پنجم تمرینات ریاضی مهندسی

مسئله اشترم-لیوویل هر یک از متغیرهای مکان مسائل زیر را به دست آورید.

$$1. \quad u_{tt} + \gamma u_t - u_{xx} + \gamma u_x + u = xt, \quad 0 \leq x \leq \lambda, t \geq 0 \\ u(x, 0) = 0, \quad u_t(x, 0) = 0, \\ u(0, t) = e^t, \quad u(\lambda, t) = t$$

$$2. \quad u_t - \gamma u_{xx} - u_{txx} = xt, \quad 0 \leq x \leq \lambda, t \geq 0 \\ u(x, 0) = x^\gamma, \quad u(0, t) = t, \quad u_x(\lambda, t) = t^\gamma$$

$$3. \quad u_t = u_{xx} + \gamma u_{yy} + u + xyt, \quad 0 \leq x, y \leq \lambda, t \geq 0 \\ u(x, y, 0) = xy, \\ u(0, y, t) = yt^\gamma, \quad u_x(\lambda, y, t) = y^\gamma t, \\ u_y(x, 0, t) = x^\gamma t^\gamma, \quad u_y(x, \lambda, t) = xt$$

$$4. \quad r^\gamma u_{rr} + ru_r + u_{\theta\theta} = r \sin \theta, \quad 0 \leq \theta \leq \pi, \lambda \leq r \leq \gamma \\ u(r, 0) = r, \quad u_\theta(r, \pi) = 0, \quad u(\lambda, \theta) = \theta, \quad u(\gamma, \theta) = 0$$

$$5. \quad u_{tt} = u_{xx} + \gamma u_{yy} + \gamma u_y + u_{zz}, \quad 0 \leq x, y, z \leq \lambda, t \geq 0 \\ u(x, y, z, 0) = xy^\gamma, \quad u_t(x, y, z, 0) = yz^\gamma, \\ u(0, y, z, t) = zt, \quad u_x(\lambda, y, z, t) = yt^\gamma, \\ u_y(x, 0, z, t) = xt, \quad u_y(x, \lambda, z, t) = zt, \\ u_z(x, y, 0, t) = yt, \quad u(x, y, \lambda, t) = xt^\gamma$$

$$6. \quad u_{tt} + \gamma u_t + u_{txx} + u_{xxxx} = xt^\gamma, \quad 0 \leq x \leq \lambda, t \geq 0 \\ u(x, 0) = x^\gamma, \quad u_t(x, 0) = x, \\ u_x(0, t) = t^\gamma, \quad u(\lambda, t) = t, \quad u_{xxx}(0, t) = \lambda, \quad u_{xx}(\lambda, t) = t$$